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CONSTRUCTION AND TESTING OF A RAREFIED GAS FLOW FACILITY

Arnold H. Henderson, LT, USN

Aeronautical Engineer 1967

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CONSTRUCTION AND TESTING OF A RAREFIED GAS FLOW FACILITY

Thesis by

Arnold H. Henderson

Lieutenant, United States Navy

In Partial Fulfillment of the Requirements
For the Degree of
Aeronautical Engineer

California Institute of Technology
Pasadena, California
1967
(Submitted September 22, 1966)

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ACKNOWLEDGMENTS

My sincere thanks go to the United States Navy for providing the opportunity for post-graduate education; to Dr. Edward E. Zukoski for initiation of the project and consultative assistance; to Dr. Frank E. Marble for research assistance; to Mr. Frank T. Linton for assistance with facility construction and thesis drawings; to Mrs. Roberta Duffy for typing of the final manuscript; to my wife, Ardis, for encouragement during the blackest hours; and to my fearless leader and fellow conspirator, LCDR James A. McGill, for his major contribution to the project.

Thanks are also due to Dr. Hans W. Liepmann for the use of the vacuum vessel which he designed and used to investigate orifice flow.

ABSTRACT

A low-density gasdynamic facility, suitable for measurement of mass flow through small (approximately 1 mm diameter) nozzles, tubes, and orifices, was designed and constructed. The system is capable of producing and measuring mass flows at 100:1 upstream/downstream pressure ratios for the entire spectrum from continuum to free molecular flow.

Results of preliminary tests on an orifice are shown and compared with the work of previous investigators. The regime of transition flow is apparently shown to extend to higher Knudsen numbers than previously postulated. The ratio of actual mass flow to theoretical free-molecule mass flow is shown to increase smoothly from a limiting value of 1.0 at high Knudsen numbers to an apparent limiting value at very low continuum-flow Knudsen numbers.

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LIST OF SYMBOLS

Kn	Knudsen number, ratio of mean free path to characteristic length of flow channel $= \lambda/D$
λ	mean free path, cm
D	diameter, cm
μ	viscosity, poise
ρ	density, g/cm^3
V_a	mean molecular speed, cm/sec
R	gas constant per gram $= \mathcal{R}/M_w$
$\mathcal{R}$	universal gas constant $= 8.314 \times 10^7 \dfrac{\text{dyne-cm}}{\text{g-}^\circ\text{K}}$
M_w	molecular weight, g/mole
T	absolute temperature, $^\circ K$
P	pressure, $dyne/cm^2$
γ	ratio of specific heats
a	speed of sound
Re	Reynolds number
M	Mach number
$\dot{m}$	mass flow rate, g/sec
A	cross-sectional area, cm^2
H	perimeter of cross section, cm
L	length (of tube or nozzle), cm
κ	Clausing factor
Γ'	$= \sqrt{\gamma(2/\gamma+1)^{(\gamma+1)/(\gamma-1)}}$
α	orifice discharge coefficient
Q	volume flow rate, cm^3/sec

Γ	non-dimensionalized mass flow rate = $\dot{m}/(\rho_c WA)$
W	characteristic velocity or orifice flow = $\left((P_c - P_e)/\rho_c\right)^{\frac{1}{2}}$
Γ_K	non-dimensionalized mass flow at the free molecular limit = $\kappa/\sqrt{2\pi}$

Subscripts

c	stagnation chamber conditions
e	exhaust chamber conditions

I. INTRODUCTION

Considerable interest has been generated recently in the problem of flow through nozzles at very low chamber pressures, inasmuch as low sustained thrusts are often desired to control spacecraft orientation or to make small orbital changes. Rocket motors have in fact been designed to give thrusts on the order of a micropound. Although, understandably, little data are available on the actual measured thrust and mass flow of these motors, it is estimated that gas flow through the nozzle falls in a regime of flow which is intermediate between continuum gas flow and free molecular flow.

It was thus determined to design and construct a low-density gasdynamic facility suitable for measurement of mass flow through orifices, short tubes, and nozzles in this regime.

II. THEORETICAL BACKGROUND

The characteristics of gas flow through a bounded space vary markedly with the density of the gas. Knudsen was among the first to investigate, both theoretically and experimentally, the case wherein the gas is rarefied to the extent that the average distance traveled by each molecule between collisions with other molecules is of the same order as, or greater than, the lateral dimensions of the flow channel. Knudsen in fact defined the ratio of molecular mean free path to some characteristic channel dimension (usually the diameter for circular cross sections) as a criterion for determining the nature of the flow. Utilizing this Knudsen number, we can define flow regimes ranging from ordinary continuum flow, where the gas acts as a fluid matrix, to the aforementioned free molecular flow where the gas exhibits all the characteristics of its coarse molecular structure.

Tsien[1] and others proposed the names for the various regimes and, following Schaaf and Chambre[2] and Dushman[3], we can divide the regimes roughly by Knudsen number as follows:

continuum flow	$Kn < .01$
slip flow	$.01 < Kn < .1$
transition flow	$.1 < Kn < 1.0$
free molecular flow	$Kn > 5$

The Knudsen number has been defined as the ratio of mean free path to flow channel diameter, i.e.,

$$Kn = \lambda/D \; . \tag{1}$$

From the kinetic theory of gases[2], we obtain:

$$\lambda = \frac{2\mu}{\rho V_a} \tag{2}$$

where:

μ = viscosity,

$\rho = P/RT$ = density,

$V_a = \sqrt{8RT/\pi}$ = mean molecular speed,

R = gas constant per gram,

T = absolute temperature, and

P = pressure.

Substituting for ρ and V_a :

$$\lambda = \frac{2\mu\sqrt{\pi}\, RT}{P\sqrt{8RT}} = \frac{\mu\pi}{P}\sqrt{\frac{RT}{2\pi}}$$

or

$$\lambda \cong \frac{16\mu}{5P_cD}\sqrt{\frac{RT}{2\pi}} \tag{3}$$

after Sreekanth[4].

Then Knudsen number, based on stagnation chamber conditions, is:

$$Kn = \frac{16\mu}{5P_cD}\sqrt{\frac{RT_c}{2\pi}} \ . \tag{4}$$

The mean molecular speed is related to the speed of sound as:

$$a = V_a\sqrt{\pi\gamma/8} \tag{5}$$

where:

γ = ratio of specific heats,

a = speed of sound.

Then:

$$\lambda = \frac{2\mu}{\rho V_a} = \frac{2\mu}{\rho a}\sqrt{\pi\gamma/8} \tag{6}$$

$$Kn = \lambda/D = 1.26\sqrt{\gamma}\frac{\mu}{\rho a D} \tag{7}$$

The Reynolds number based on the same dimension, D , is:

$$Re = \rho VD/\mu = \rho aMD/\mu$$

$$Re/M = \rho aD/\mu \tag{8}$$

where M = Mach number.

Substituting this expression in equation (7) gives the fundamental relation:

$$Kn = 1.26\sqrt{\gamma}\,M/Re \ . \tag{9}$$

The most significant phenomenon of slip flow is the fact that the layer of gas adjacent to the wall can no longer be considered at rest as in continuum flow, but has some finite velocity. Thus, the equations for continuum flow, such as the Navier-Stokes equations, can still be used in slip flow with certain semi-empirical modifications.

Free molecule flow is quite rigorously analyzed by use of the kinetic theory of gases. The assumption can successfully be made that intermolecular collisions are unimportant compared with molecule-wall collisions. Then the fluxes of incident molecules and reflected molecules can be treated independently since there is little chance that a reflected molecule will collide with incoming molecules.

However, in the transition regime, density of the gas is such that the mean free path is of the same order as, or less than, the flow conduit diameter. In this case, intermolecular collisions cannot be ignored, but neither does the gas very closely resemble a

continuous fluid. The analysis of the flow becomes quite complicated and no really satisfactory theory has yet been devised to describe flow in the transition regime. The need for more experimental data in this area has been noted by Sherman[5] and others.

The flow regimes delineated above of course grade smoothly into each other such that any line of demarcation between regimes is somewhat arbitrary. However, examination of the theories and analytic techniques generally accepted or proposed by various authors for flow through orifices, short tubes, and nozzles in each regime will aid in defining the scope of the present investigation. Since determination of mass flow rate through the above conduits is the objective of the low-density gasdynamic facility, analytic equations for mass flow rate will be the objective of the following discussion.

A. Free Molecular Flow

Present[6] and Dushman[3] give the equation which Knudsen deduced from kinetic theory for mass flow through a conduit of varying cross section. Knudsen considered the walls to be of such roughness that the reflections were completely diffuse, a quite practical assumption since surfaces of the smoothness of a crystal cleavage plane or better are required for specular reflections[6]. Knudsen's formula is:

$$\dot{m} = \frac{\frac{4}{3} V_a (P_c - P_e)}{RT \int_0^L \frac{H}{A^2} dL} \tag{10}$$

where:

V_a = $\sqrt{8RT/\pi}$ = mean molecular speed

P_c = stagnation chamber pressure

P_e = exhaust chamber or exit pressure

A = varying cross section

H = perimeter of cross section

L = length along nozzle

T = absolute temperature

R = gas constant for one gram

Substituting for V_a :

$$\dot{m} = \frac{\frac{4}{3}\sqrt{8RT/\pi}\,(P_c - P_e)}{RT\int_0^L \frac{H}{A^2}\,dL} = \frac{4}{3}\sqrt{8/\pi RT}\;\frac{(P_c - P_e)}{\int_0^L \frac{H}{A^2}\,dL} \quad . \qquad (11)$$

The equation (11) is readily reducible for any nozzle where the cross-sectional area and perimeter are known functions of L . For a circular, constant-area tube, equation (11) reduces to:

$$\dot{m} = \frac{4}{3}\sqrt{8/\pi RT}\;\frac{(P_c - P_e)}{4\pi DL/A\pi D^2} = \frac{4D}{3L}\;\frac{A(P_c - P_e)}{\sqrt{2\pi RT}} \qquad (12)$$

where D = tube diameter. We at once perceive the anomaly that mass flow through a zero-length orifice becomes infinite if we attempt to use equation (12). The theoretical work of P. Clausing[7] some years after Knudsen on the problem of free molecular flow through short tubes and orifices produced the well-known Clausing factor, which enters as:

$$m = \frac{\varkappa A(P_c - P_e)}{\sqrt{2\pi RT}} \qquad (13)$$

where $\varkappa$ = Clausing factor.

Clausing showed that κ is 1.0 for a zero-length orifice and approaches 4D/3L for large ratios of L/D, i.e., long tubes. Dushman[3] gives a plot of Clausing factor versus L/D for L/D ratios from 0 to 23, and tables for L/D ratios up to 500.

B. Continuum Flow

At high Reynolds numbers, continuum fluid flow through short tubes and nozzles is effected by boundary layers to only a small degree, and that through an orifice to an even smaller degree because of the very small passage length. However, another problem arises in the computation of continuum mass flow through an orifice when the pressure ratio across the orifice is high enough for transonic flow velocities. As Liepmann[8] points out, an S-shaped sonic line exists for high pressure ratios across an orifice. Analysis by the method of characteristics in the hodograph plane shows surprisingly high pressure ratios ($P_c/P_e = 58.3$ for $\gamma = 5/3$, for instance) required for choked flow, and indicates a mass flow attenuation from the one-dimensional isentropic value since the flow at the plane of the orifice is not sonic over the whole area. The one-dimensional analysis gives:

$$\begin{aligned} \dot{m} &= \rho A V = \rho A a M \\ &= \frac{P}{RT}\sqrt{\gamma R T}\,A \qquad \text{for } M = 1.0 \\ &= P_c \frac{P}{P_c}\sqrt{\frac{\gamma}{RT_c \frac{T}{T_c}}}\,A \\ &= \frac{P_c A}{T_c}\,\frac{1}{R}\sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} \end{aligned}$$

$$\dot{m} = \Gamma' \frac{P_c A}{\sqrt{RT_c}} \tag{14}$$

where:

P_c = stagnation chamber pressure

T_c = stagnation chamber temperature

R = gas constant per gram

A = cross-sectional area of the orifice

$\Gamma' = \sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$

which is a familiar result. However, the S-shaped sonic line requires the inclusion of an additional factor to modify Γ', which will also be a function of γ, i.e.,

$$\dot{m} = \alpha\Gamma' \frac{P_c A}{\sqrt{RT_c}} \quad . \tag{15}$$

The factor α is normally obtained from experimental data for a given orifice, but Frankl[9] has made a numerical computation for gas flowing through a two-dimensional slit. He obtains $\alpha = .85$ for a gas with $\gamma = 1.4$. Liepmann[8] indicates that the value of α for an axisymmetric orifice is probably little different from that for a slit.

Analysis of continuum flow through a nozzle is highly dependent on the Mach number. If Mach number is 1.0 or above, the Reynolds number necessary for the flow to be considered in the continuum regime is above 100 and the flow can be analyzed either by one-dimensional isentropic gasdynamics with thin boundary layer modifications or by the method of characteristics, considering shock and expansion waves.

However, at Mach numbers well below 1.0, the Reynolds number can drop below 100 and the flow still be in the continuum regime. Then we get so-called Stokes "creeping flow," where the inertia terms in the Navier-Stokes equations can be neglected compared to the viscous terms. Roscoe[10] has shown that creeping mass flow through an orifice is related to the pressure drop as:

$$\dot{m} = \frac{D^3\rho}{24\mu} (P_c - P_e) \tag{16}$$

where:

D = orifice diameter,

P_c = upstream pressure,

P_e = downstream pressure,

μ = viscosity, and

ρ = gas density.

Weissberg[11] has obtained a similar expression for short tubes:

$$m = \frac{D^3\rho}{24\mu} \frac{(P_c - P_e)}{\left(1 + \frac{16}{3\pi}\frac{L}{D}\right)} \tag{17}$$

where L = length of tube.

Doctor F. E. Marble, California Institute of Technology, has done a theoretical analysis in an unpublished paper entitled "Flow of Gas through a Nozzle at Very Low Reynolds Number." By the definition of the present investigation, for a throat Mach number of 1.0, Marble's analysis for throat Reynolds numbers below 100 would fall into the category of low continuum, bordering on slip flow. To quote a portion of the paper,

> "... For these low Reynolds number nozzles, the viscous effects are no longer confined to small regions near

> the walls but extend over the entire nozzle cross section. ... The following analysis attempts to treat the low Reynolds number nozzle problem by modifying the formulation to include the viscous stresses associated with wall shear but neglecting the normal viscous stress associated with acceleration along the flow direction. It is assumed that the flow is dissipative but locally adiabatic..."

Marble integrates the continuum equations of continuity, momentum (including a term associated with viscous stress at the nozzle wall), and the First Law of Thermodynamics. His result for mass flow through the nozzle is:

$$\dot{m} = \frac{A\rho_c \eta^*\left(1 - \frac{\gamma-1}{2}\eta^{*2}\right)^{\frac{1}{\gamma-1}}}{\exp \frac{\gamma}{\gamma-1}\,\frac{S_{(\eta^*)}-S_c}{C_p}} \qquad (18)$$

where:

A = area of throat

ρ_c = stagnation chamber density

a_c = stagnation chamber speed of sound

$\eta^* = u^*/a_c$

u^* = average fluid velocity at the throat

γ = ratio of specific heats

$S_{(\eta^*)}$ = entropy at the throat

S_c = entropy in the stagnation chamber

C_p = specific heat at constant pressure

$$\frac{S_{(\eta^*)}-S_c}{C_p} = \frac{L}{\lambda}\left(\frac{1}{1-\frac{\gamma-1}{2}\eta^{*2}}\right)$$

λ = a characteristic length related to viscous stresses

$= \frac{\dot{m}}{2\pi\phi\mu}$

μ = viscosity

ϕ = numerical constant depending upon the assumed shape of the velocity profile

$\cong$ 2 for reasonable profile shape

We can also consider the Hagen-Poiseuille equation for flow through tubes[3]:

$$\dot{m} = \frac{D^4 \pi}{128 \mu L} \frac{(P_c + P_e)}{2} \frac{(P_c - P_e)}{RT} \tag{19}$$

but with the restrictions that the flow is: (1) laminar; (2) incompressible ($M \leq .33$); (3) fully developed (tube completely filled by the viscous boundary layer); and (4) the flow velocity at the wall is zero.

C. Transition and Slip Flow

The Poiseuille formula for long circular tubes with slip boundary conditions is given by Present[6] as:

$$\dot{m} = \left[\frac{\pi D^4}{128 \mu RT} \left(\frac{P_c + P_e}{2} \right) (P_c - P_e) + \frac{.519 D^3}{V_a} (P_c - P_e) \right] \frac{1}{L} \quad . \tag{20}$$

Sreekanth[4] discovered from his data that the above equation multiplied by the factor L/(L+D) predicted mass flow fairly well for short tubes and orifices in the transition regime. His modified Poiseuille equation is:

$$\dot{m} = \left[\frac{\pi D^4}{128 \mu RT} \left(\frac{P_c + P_e}{2} \right) (P_c - P_e) + \frac{.519 D^3}{\sqrt{\frac{8RT}{\pi}}} (P_c - P_e) \right] \frac{1}{L\left(1 + \frac{D}{L}\right)} . \tag{21}$$

Dividing by the cross-sectional area and rearranging:

$$\frac{\dot{m}}{A} = \frac{P_c - P_e}{2\left(1 + \frac{L}{D}\right)\sqrt{2\pi RT}} \left[\frac{\sqrt{2\pi}\, D P_c \left(1 + \frac{P_e}{P_c}\right)}{32 \mu \sqrt{RT}} + 2.076 \right]$$

$$\frac{\dot{m}}{A} = \frac{P_c - P_e}{2(1 + L/D)\sqrt{2\pi RT}} \left[\frac{1 + P_e/P_c}{10\ Kn} + 2.076\right] \qquad (22)$$

where:

D = tube diameter

μ = viscosity

R = gas constant per gram

T = absolute gas temperature

P_c = stagnation chamber pressure

P_e = exit pressure

L = tube length

V_a = mean molecular speed = $8RT/\pi$

We have theoretical free molecular mass flow as (equation 13):

$$\frac{\dot{m}_{fm}}{A} = \frac{\kappa(P_c - P_e)}{\sqrt{2\pi RT}}$$

and we obtain the ratio of mass flow as given by equation (22) to the theoretical free-molecule flow value :

$$\frac{\dot{m}}{\dot{m}_{fm}} = \frac{1}{2\kappa(1 + L/D)} \left[\frac{1 + P_e/P_c}{10\ Kn} + 2.076\right] \qquad (23)$$

as was done by Sreekanth[4].

There is also the theoretical analysis of Narasimha[12] for orifice flow at high Knudsen numbers. He obtains:

$$\frac{\dot{m}}{\dot{m}_{fm}} = 1 + .25\ R/\lambda_1 \qquad (24)$$

where, in this case, R = radius of the orifice and λ_1 = mean free path under upstream conditions, or, in terms of Knudsen number, $Kn = \lambda/D$:

$$\frac{\dot{m}}{\dot{m}_{fm}} = 1 + .125/Kn \ . \qquad (25)$$

The fit of this formula with Liepmann's[8] orifice data is fair (as shown by Narasimha's own plot) from free molecular flow to a Knudsen number of about 1.0 , but it begins to deviate rapidly at lower Knudsen numbers.

Numerical solutions of the Boltzmann equation in the transition regime by several authors are discussed by Willis[13], but none seem to be directly applicable to flow through enclosed channels.

III. DESCRIPTION OF LOW-DENSITY GASDYNAMIC FACILITY

A. General

A photograph of the apparatus is shown in Figure 1, and systems diagrams are shown in Figures 2, 3, and 4. The facility was designed and built as a continuous flow, open circuit system with provision for control and measurement of flow over a stagnation pressure range of 1 micron of mercury to one half atmosphere. Pump capacities were selected to maintain a pressure ratio of at least 100:1 across the test profile. The downstream pressure can, however, be roughly throttled by use of large vacuum gate valves between the tank exhaust chamber and each of the pumps.

Volume flow rate, pressure, and temperature of the test gas are measured at slightly above atmospheric pressure before the gas is throttled into the vacuum tank stagnation chamber. Pressure and temperature of the gas in the stagnation chamber and gas pressure in the exhaust chamber can also be accurately measured.

B. Pumping System

Figure 4 is a schematic diagram of the pumping system. On the basis of nominal test profile diameters of 1 millimeter, a Consolidated Vacuum Corporation PMCS-2C 2-inch oil diffusion pump was selected to maintain at least a 100:1 pressure ratio across the test profile for exhaust chamber pressures up to 10 microns of mercury. The backing pump for the diffusion pump is a nominal 7 cubic feet per minute (CFM) Central Scientific Company "Hypervac 25", slightly larger than the 5 CFM backing required for maximum ca-

pacity pumping by the PMCS-2C diffusion pump.

Above 10 microns Hg exhaust chamber pressure, the diffusion pump can be isolated from the system by closing a Temescal 2-inch vacuum gate valve. In this manner, the diffusion pump oil can be continuously maintained at operating temperature, precluding delays for warmup and shutdown of the diffusion pump.

Pumping above 10 microns Hg exhaust chamber pressure is accomplished by a 300 CFM Stokes 412H rotary vacuum pump. When desired, this pump can also be isolated from the system by closing a Temescal 4-inch vacuum gate valve. Pumping characteristics of the Stokes 412H pump and the PMCS-2C diffusion pump are contained in Figures 5 and 6, respectively.

To the exhaust end of the horizontally-mounted cylindrical vacuum test tank is welded a flanged, 4-inch steel pipe of approximately four inches length. A horizontal 4-inch diameter aluminum pipe with a 2-inch vertical tee is bolted to the flanged end of the steel pipe from the tank, with a blank flange containing an exhaust chamber pressure tap inserted between the two pipe flanges. At the opposite end of the horizontal 4-inch diameter aluminum pipe is a standard ASA 150 # aluminum flange connecting to a 4-inch Temescal Series 5000 vacuum gate valve. Joined to the other side of the valve, by another ASA 150# aluminum flange, is a 6-inch diameter aluminum reversed double elbow with a vertical fall of twelve inches. This elbow is to prevent Stokes vacuum pump oil from backstreaming into the system. Between the flanged bottom end of the elbow and the flanged pipe inlet to the Stokes pump is a steel bellows designed to

isolate pump vibration from the system.

All joints, with the exception of those at the two ends of the bellows, are sealed by O-rings with a groove in one flange surface. The flanged connections at either end of the bellows are sealed by "Con-O-Rings," flat, concentric metal rings with an elastomer O-ring in the middle.

A 2-inch Temescal vacuum gate valve is connected to a lightweight aluminum flange at the bottom of the previously mentioned 2-inch vertical tee by six cap screws and is sealed with an elastomer O-ring. The Consolidated Vacuum Corporation PMCS-2C metal-oil fractionating diffusion pump is mounted vertically below the 2-inch valve, secured with six cap screws and sealed with a 2-inch Con-O-Ring.

The outlet of the diffusion pump is joined via a flexible coupling of ordinary Tygon flexible plastic tubing to a 1-inch diameter copper pipe which, in turn, is soldered to the inlet flange of the Hypervac 25 backing pump. Hose clamps seal the Tygon tubing to the diffusion pump outlet and copper pipe.

Pump-down time of the system from atmospheric pressure is shown in Figure 10.

C. Test Chamber

The vacuum test vessel is a heavy, cylindrical mild steel tank, shown schematically in Figure 3, which is separated into stagnation and exhaust chambers by an internal bulkhead which contains a removable O-ring-sealed circular plate. In the center of the plate is

drilled the desired test profile. Internal diameter of the tank is about 40 centimeters and upstream and downstream chambers are approximately 40 cm and 50 cm long, respectively. Since the ratio of tank diameter to the 1 mm test profile is 400:1 , the test profile can be considered as an orifice or nozzle in an infinite plane wall.

An internal bypass around the dividing bulkhead, which is controlled by a vacuum valve outside the downstream end of the tank, can be opened to allow rapid pump-down of the stagnation chamber when a test nozzle is in place.

Either end of the tank is secured to a rectangular stainless steel plate, 21" x 21" x 3/4" on the upstream end and 24" x 24" x 3/4" on the downstream end, by 16 cap screws. Structural support is provided at each of the four corners by 39-inch long, 5/8-inch bolts which are threaded into the downstream stainless steel plate and extend through the upstream plate, secured by adjustable nuts on either side of the upstream plate.

Two 1-inch pipe pressure taps (or gas entry lines) extend from the stagnation chamber through the exhaust chamber and downstream stainless steel end plate, terminating in flange connections outside the tank. All cutouts in the tank internal bulkhead and downstream end plate for passage of these pipes are vacuum welded to the pipes.

A removable circular plate allows access to the tank through the upstream-end stainless steel end plate. The access plate is secured by six flat-headed machine screws and sealed with an elastomer O-ring. Two threaded $\frac{1}{2}$-inch diameter ports in the access plate are

available as pressure/temperature taps or test-gas entry ports. The present installation utilizes the centerline port for test gas entry and the off-center port for introduction of a precision mercury-in-glass temperature measuring device into the stagnation chamber. A lead-off line from the thermometer tube goes also to a couple of pressure measurement instruments which are described in the section titled Vacuum Pressure Measurement.

The tank is supported to the desired height by a wooden frame of 4" x 4" support members and 2" x 4" longitudinal connectors.

D. Gas Flow and Measurement System

Source of the test gas is a standard 2000-lb gas bottle, research grade, of purity 99.99 per cent or better.

Pressure reduction is accomplished by two regulators in series. The high pressure regulator is a Matheson Model 2 with delivery pressure from 25 to 650 psig. Low pressure regulation is accomplished by a Matheson Model 70B regulator with delivery pressure of 3 to 15 inches water column.

From the low pressure regulator, the gas flows through a liquid nitrogen cold trap for removal of any lingering impurities. The cold trap reservoir is a standard Dewar wide-mouth vacuum flask packed in insulating material inside a metal container. The glass cold trap is a standard Pyrex double-tube trap, 30 cm long, with 5/8" diameter top and side connections, manufactured by Greiner Glassblowing Laboratories.

A constant temperature bath returns the gas to room tempera-

ture after the cold trap. The bath consists of a 10-gallon Pyrex jar filled with water, an electrical resistance-type heating element, a temperature regulator with a precision temperature-setting thermometer, and a circulating propeller powered by a small electric motor. Residence time of the gas within the bath is increased by ten 1-ft diameter coils of the 3/8" diameter copper tube feed line.

After the bath, but prior to flow measurement, pressure and temperature taps are led off from the main flow line. Temperature is measured by a precision mercury thermometer, an ASTM Saybolt viscosity thermometer, No. 17F, with a range of 66 to 80 degrees Fahrenheit. It can be read to ± .02 degrees Fahrenheit.

The pressure line connects to a Burton dial gauge, Model No. 2311-001, with a range of 0 - 15 pounds per square inch absolute pressure, and a Wallace and Tiernan precision mercury manometer, Model FA135, with a range of 0 - 800 millimeters of mercury. The dial gauge can be read to ± .01 psia and is accurate to $\frac{1}{2}$ per cent of full scale.

The flow measuring system is shown in Figure 2. Flow rates of about 2.0 standard cc/sec and above are measured by four Brooks E/C Purge Meters: (1) R-2-15-AA with stainless steel float, range .4 - 4.3 standard cc/sec at 760 mm Hg pressure and 70°F ; (2) R-2-25-D with sapphire float, range 1.0 - 10.9 standard cc/sec; (3) R-2-25-A with sapphire float, range 4.0 - 36.0 standard cc/sec; and (4) R-2-25-B with sapphire float, range 10 - 108 standard cc/sec. The flow rates quoted are for dry helium as determined by a water displacement calibration of the system as installed. Figure 7 shows the

calibration curves for all four meters. The extensive overlap was planned to allow use of each meter only in the upper one-half of its range where it is most accurate.

The governing equation for the volume flow through the so-called "rotameters" described above is, from ref. 14:

$$Q = A_w C \left[\frac{2gV_f(\rho_f - \rho_w)}{A_f \rho_w} \right]^{\frac{1}{2}} \tag{26}$$

where:

Q = volume rate of flow

V_f = volume of float

g = gravitational constant

ρ_f = float density

ρ_w = fluid density

A_f = area of float

C = discharge coefficient, depending on particular test fluid (varies slightly with viscosity)

$A_w = \frac{\pi}{a} [(D+by)^2 - d^2]$ = area of annular orifice

D = effective diameter of tube depending on position of float

b = change in tube diameter per unit change in height

d = maximum diameter of float

y = height of float above zero position

The rotameters should be recalibrated for each different test gas used, because of the change in discharge coefficient. However, design of the rotameter is such that C may be considered a constant for small variations in other test conditions.

In the case of a gas, ρ_w is negligible in comparison with ρ_f

and the volume flow rate can be approximated as:

$$Q = A_w C \left[\frac{2gV_f \rho_f}{A_f \rho_w} \right]^{\frac{1}{2}} \qquad (27)$$

The flow rate at any test condition is:

$$Q = Q_c \left(\frac{\rho_{w_c}}{\rho_w} \right)^{\frac{1}{2}} = Q_c \left(\frac{P_c}{P} \frac{T}{T_c} \right)^{\frac{1}{2}} \qquad (28)$$

where the subscript c is the standard flow rate corresponding to a given scale reading as determined at calibration conditions. Mass flow rate is then simply:

$$\dot{m} = Q\rho_w = Q_c (\rho_{w_c} \rho_w)^{\frac{1}{2}} \ . \qquad (29)$$

Flow rates below 2.0 standard cc/sec are measured by use of a Brooks "Vol-U-Meter," a primary standard calibrator for flow meters. The Vol-U-Meter is a precision-bored, constant-inner-diameter borosilicate tube, 30 inches in length, with a cylindrical polyvinylchloride (PVC) floating piston inside. The hollow upper portion of the PVC piston is filled with mercury which is forced out through a small radial hole into a perimeter groove by turning a set screw in the top of the piston. The mercury in the piston groove forms a gas-tight mercury "O-ring" seal between the portions of the tube above and below the piston position. A stainless steel scale, graduated from 0 to 25 cubic centimeters, is mounted alongside the borosilicate tube. Precision of the tube is .2 per cent of indicated volume as shown by the scale. The tube is mounted in a modified aluminum flow meter case and is sealed top and bottom by elastomer gaskets.

With reference to Figure 2, the Vol-U-Meter bypass valve is open while the flow stabilizes at a particular variable leak setting. To measure a flow rate, the bypass is closed, which causes the piston to move up the tube. Volume displacement rate of the gas is determined by timing the piston travel between two graduations on the scale alongside the tube. Since temperature measurements upstream of the Vol-U-Meter and in the tank stagnation chamber showed negligible difference for most tests, the temperature of the gas in the Vol-U-Meter was taken as the gas input temperature. Gas pressure immediately downstream of the Vol-U-Meter was measured by a pressure tap from the flow line to a Wallace and Tiernan Model FA-233111 dual range dial gauge (0 - 25 psig and 25 - 50 psig). Thus, from measured volume flow rate, pressure, and temperature, mass flow rate through the Vol-U-Meter was readily determined.

The pressure drop across the Vol-U-Meter piston was determined to be approximately 1 inch of water column or less than .3 per cent of the absolute gas pressure at that point. However, to preclude even this small error, and to provide quicker response of the piston at low flow rates, the test gas was routed through one of the rotameters, with the rotameter inlet needle valve set to provide a pressure drop equivalent to the weight of the Vol-U-Meter piston. Then, simultaneously with the closing of the Vol-U-Meter bypass valve, the rotameter bypass valve was opened and the pressure drop quickly migrated downstream to the Vol-U-Meter piston.

Usable range of the Vol-U-Meter is about .003 cc/sec to 2.5 cc/sec. In actual fact, the lower limit is a function only of the

stamina and perseverance of the operator, since the positive displacement piston would measure 1 cc/year flow rate, if pressure and temperature conditions could be successfully averaged for the test period. The upper rate is an accuracy limitation due to the reaction time of the operator. Care must also be exercised at higher flow rates to stop the piston prior to the top of the tube and to lower it gently, else the mercury "O-ring" will break and leave mercury droplets on the tube walls. This necessitates the tedious task of disassembling the Vol-U-Meter, cleaning the tube and piston, and resetting the mercury "O-ring."

Downstream of the flow measurement devices, the flow passes through, and is ultimately controlled by, a Granville-Phillips Series 203 Variable Leak. Specifications state conductance is variable from 10^{-10} standard cc/sec to 100 cc/sec with 1 atmosphere pressure differential across the valve. The valve handle is connected to a counter which increases one number for each 1/10 handle turn. With the clutch properly adjusted, the handle slips at a counter reading of 10, corresponding to gas-tight shutoff (conductance no greater than 10^{-13} cc/sec) and is fully open at a counter reading of 270. The higher flow rates are quite reproducible at given counter settings, but the hysteresis inherent in all mechanical devices precludes anything more than approximate reproducibility at low flow rates. Figure 8 is a reasonably accurate plot of Variable Leak counter setting versus stagnation chamber pressure from data obtained during tests on the present system, using helium as the test gas and an orifice for the test profile.

Entry of the test gas into the stagnation chamber of the

vacuum tank is through the stainless steel access plate of the upstream end via a $\frac{1}{2}$-inch centerline port. Tests were made both with the gas entering parallel to the flow direction through the test profile and turned 90° by a $\frac{1}{2}$-inch copper elbow. No difference in results was observed, indicating little likelihood that straight-through streamlines near the centerline increased the mass flow through the test profile over and above that due to the measured conditions in the stagnation chamber.

It would also be possible to introduce the test gas through one of the off-centerline 1-inch pipes which extend from the downstream end of the tank through the exhaust chamber and dividing bulkhead into the stagnation chamber. This would completely eliminate any lingering doubts about straight-through flow effects, since the flow direction into the chamber would be opposite to the flow direction through the test profile.

Stagnation chamber temperature was measured by a precision mercury-in-glass thermometer, an ASTM Saybolt viscosity thermometer, No. 17F, range 66 - 80 degrees Fahrenheit, which could be read to ± .02°F. The thermometer was introduced into the stagnation chamber through an off-center $\frac{1}{2}$-inch port in the access plate of the upstream stainless-steel end plate. Sealing around the glass thermometer tube was accomplished by use of a gauge tube connector with neoprene bushing.

Accuracy of this method of temperature measurement was somewhat questionable at very low stagnation chamber pressures, but the constancy of temperature between the input measurement and this

stagnation chamber measurement at higher pressures made it reasonable to assume such was also the case at low stagnation chamber pressures.

E. Test Plate

The test plate occupies the center of the dividing bulkhead which separates the stagnation and exhaust chambers of the tank. Figure 9 shows the plate with a typical test profile. The plate is secured to a recessed cutout in the tank dividing bulkhead by three flat-head machine screws equally spaced near the perimeter and is sealed with an elastomer O-ring. The flat upstream side of the test plate is flush with the bulkhead, making the test profile an aperture in an essentially infinite plane wall, since the ratio of tank diameter to test profile diameter is approximately 400:1 .

F. Vacuum Pressure Measurement

The sole exhaust chamber pressure tap is a 3/8-inch copper tube which extends from a blank 4-inch aluminum flange at the tank outlet into the exhaust chamber, where it bends 90° to the direction of the gas flow. To the outside of the hole in the blank flange is connected a 3/8-inch copper tube line to a mercury McLeod gauge. A blind tee from this line terminates in a Consolidated Vacuum Corp. GTC-004 Thermocouple Gauge Tube, the sensing element of a Consolidated Vacuum Corp. Thermocouple Vacuum Pressure Gauge Type GTC-100, range 0 - 1000 microns Hg, referred to hereafter as the "thermistor" gauge.

Two upstream pressure taps are utilized. The first makes

use of a 1-inch pipe extending from outside the downstream end of the tank, through the exhaust chamber and dividing bulkhead. To the flange at the outside end of the 1-inch pipe is connected (and sealed with an elastomer O-ring) a mating flange to which is soldered a 3/8-inch copper tube that joins the line to the McLeod gauge. Shutoff valves in the upstream and downstream pressure tap lines, prior to the point where they join the single line to the McLeod gauge, make it possible to measure upstream and downstream pressures independently with the McLeod gauge. The McLeod line passes through a liquid nitrogen cold trap prior to reaching the gauge, not only to condense impurities in the test gas but also to protect the system from mercury vapor emanating from the McLeod gauge. The cold trap reservoir is a Dewar wide-mouth vacuum flask packed in insulating material inside a metal container.

The second stagnation chamber pressure tap extends from the off-center port in the access plate at the upstream end of the tank, via a 3/8-inch copper tube, to a 0 - 50 mm Hg Wallace and Tiernan absolute pressure dial gauge and a 0 - 800 mm Hg Wallace and Tiernan precision mercury manometer, Model FA-130. Both of these instruments can be isolated by Circle Seal 1/4-inch vacuum shutoff valves. The manometer was isolated from the system below 50 mm Hg stagnation chamber pressure to prevent mercury vapor contamination of the Granville-Phillips Variable Leak.

The vacuum pressure line from the stagnation chamber is connected to the top of the manometer with the mercury reservoir open to the atmosphere. The manometer mercury column height was sub-

tracted from a Central Scientific Company mercury-in-glass barometer reading to give the absolute stagnation chamber pressure. The dial gauge was set to agree with the McLeod gauge and appeared to stay in calibration over a period of two months. The dial gauge can be read to an accuracy of ± .01 mm Hg.

A blind tee from the above pressure tap line terminated in a Consolidated Vacuum Corp. GTC-004 Thermocouple Gauge Tube, the sensing element of the Consolidated Vacuum Corp. Thermocouple Vacuum Pressure Gauge, Type GTC-100. The two-channel thermistor gauge, with one channel connected to the exhaust chamber sensing element and one channel to the stagnation chamber sensing element, was calibrated periodically but was never used as more than a rough pressure indication in setting the variable leak or determining the exhaust pressure when it was necessary to switch from the diffusion pump to the Stokes 412H rotary pump. In fact, all precision pressure measurements up to 100 mm Hg were made by the McLeod gauge, and those above 100 mm were made by the mercury manometer.

The McLeod array, made by Greiner Glassblowing Laboratory, consisted of two separate gauges; a non-linear gauge with a range of 0 - 125 microns of mercury, and a "linear" gauge with three initial/final volume ratios and three respective reading tubes with ranges of 0 - 1 mm Hg, 0 - 10 mm Hg, and 0 - 100 mm Hg.

The non-linear gauge has a capture volume of 330.6 cubic centimeters which is compressed into a capillary tube 230 mm in length until the height of the mercury in the parallel tube is at the same height as the closed end of the capillary. The difference in

mercury column height is read on a mirrored scale to an accuracy of ± .1 mm. Pressure is determined as per the following analysis:

$P_1V_1 = P_2V_2$ assuming an isothermal compression

$P_1 = P_2V_2/V_1$

$P_2 = \Delta h + P_1$ (all pressures in Hg column height) but P_1 can be neglected compared to Δh

$V_2 = \Delta h\, A_c$

$$P_1 = \Delta h(\Delta h A_c)\frac{1}{V_1} = (\Delta h)^2\frac{A_c}{V_1} \qquad (30)$$

where:

P_1 = unknown pressure to be measured

P_2 = final pressure of compressed gas

Δh = differential mercury column height

A_c = area of capillary tube bore

V_1 = capture volume (known)

V_2 = final volume of compressed gas.

The "linear" gauge has a capture volume of 250.5 cm^3 which it compresses to three successively smaller volumes which are also known. A 500-mm mirrored scale is set behind each of three tubes with the zero point of the scale at the level of the corresponding compression volume calibration mark. When the mercury level is stopped at any of the three compression volume marks, the differential mercury column height is read on the scale behind the appropriate tube. Pressure is then determined as follows:

$P_1V_1 = P_{2_i}V_{2_i}$

$$P_{2_i} = \Delta h_i + P_1$$

$$P_1 V_1 = (\Delta h_i + P_1) V_{2_i}$$

$$P_1 = \Delta h_i \left(\frac{1}{V_1 / V_{2_i} - 1} \right) \tag{31}$$

where:

P_1 = unknown pressure to be measured

P_{2_i} = final pressure of compressed gas

V_1 = initial capture volume (known)

V_{2_i} = compressed volume (known)

Δh_i = differential mercury column height

i = 1, 2, or 3, corresponding to test sample compression ratio and scale to be read.

All mercury column heights were corrected to zero degrees Centigrade and standard gravity before use, except for the Wallace and Tiernan mercury manometers which are equipped with temperature-compensating scales.

A gas pressurization system was used to raise and lower the mercury level in the McLeod gauge. A nitrogen bottle, regulated by a Matheson Model 1L regulator with delivery pressure from 3 to 80 psig, is connected to a Circle Seal MV90 Series Needle Control Valve. This needle valve meters the flow of nitrogen to pressurize the volume above the mercury level in the McLeod mercury reservoir and to raise the mercury level in the gauge. Precision control of the rate of mercury level rise is accomplished merely by opening or closing the needle valve. Stopping the mercury level at any point in the

gauge can be done either by closing the needle valve to gas-tight shutoff or by closing the individual gauge shutoff valves. A vent valve in the nitrogen pressurization line is opened to lower the mercury level.

IV. RESULTS AND DISCUSSION

Preliminary tests were conducted on one nozzle profile, an orifice similar to that shown in Figure 9. Helium was chosen as the test gas by reason of its relatively low density, low molecular weight, and chemical inertness. To elaborate on the first two points, consider first equation (4):

$$Kn = \frac{16\mu}{5P_c D}\sqrt{\frac{\Re T_c}{M_w 2\pi}}$$

where M_w = molecular weight, and $\Re$ = gas constant per mole, which indicates that lower molecular weight is desirable to reach a given Knudsen number with higher stagnation chamber pressure (P_c). Higher stagnation chamber pressure helps eliminate the inaccuracies caused by leaks and outgassing.

To show the desirability of low density, consider a mass flow near the free molecular regime where we can take theoretical free molecular flow rate as a good approximation to that actually obtained. From equation (13):

$$\dot{m}_{fm} = \frac{\varkappa A(P_c - P_e)}{\sqrt{2\pi RT}} .$$

The volume flow rate is simply the mass flow rate divided by density:

$$Q = \frac{\dot{m}_{fm}}{\rho} = \frac{\varkappa A}{\rho}\,\frac{(P_c - P_e)}{\sqrt{2\pi RT}} . \qquad (32)$$

Clearly, lower density gives a higher, and therefore simpler to measure, volume flow rate for a given stagnation chamber pressure.

The preliminary tests were done in order to "shake down" the instrumentation and to ensure that the system was capable of cover-

ing the desired flow range from continuum to free molecular. A typical test procedure is covered in Appendix I.

The initial series of tests, spanning a period of about one month, gave erroneous results at high Knudsen numbers (1 to 10), indicating mass flows as much as 10 per cent below the theoretical free-molecular flow rate for the measured stagnation chamber pressure. An exhaustive series of leakage tests finally detected a leak into the stagnation chamber of about 1 micron-liter per second. A leak of this magnitude was negligible at lower Knudsen numbers and had no effect on results, but was sufficient to impair the data near the free molecular end.

Another possible source of error at low flow rates was the slight pressure drop built up across the Vol-U-Meter piston for low flow rate measurement. Although the magnitude of the pressure change in the feed line when the Vol-U-Meter bypass was closed was a fraction of one per cent, an oscillating pressure wave may have had a much greater effect on the flow rate measurement. The remedy was to route the test gas through one of the rotameters, with the rotameter inlet needle valve set to provide a pressure drop equivalent to the weight of the Vol-U-Meter piston. Then, simultaneously with the closing of the Vol-U-Meter bypass valve, the rotameter bypass valve was opened and the pressure drop quickly migrated downstream to the Vol-U-Meter piston.

Elimination of the leak produced apparently accurate data for a second series of tests. In the subsequently described presentations of data, only the results from the second series of tests are

used at high Knudsen numbers, but both sets are shown at Kn = .1 and below. Each data point at Kn = 1.0 and above required a period of five hours or more for system stabilization due to the low flow rates and low stagnation chamber pressures. The flow measurement procedure alone often consumed an hour or more for each point.

The pumping system adequately maintained at least a 100:1 pressure ratio across the .039-inch diameter (or approximately 1-millimeter diameter) orifice throughout the entire test range. Figures 5 and 6 show the diffusion pump and the Stokes 300 CFM pump both tailing off badly in capacity (from opposite ends) at the crossover point of P_e = 10 microns of mercury. However, no difficulty was experienced in maintaining at least a 100:1 pressure ratio even at the crossover exhaust chamber pressure.

Figure 11 shows the test data plotted as Liepmann's[8] Γ/Γ_K versus $1/Re$, where:

$$\Gamma_K = \kappa/\sqrt{2\pi} = \text{theoretical free molecular value,}$$

$$\Gamma = \frac{\dot{m}}{\rho_c W A} = \frac{\dot{m}}{\rho_c \left(\frac{P_c - P_e}{\rho_c}\right)^{\frac{1}{2}} A} = \frac{\dot{m}\sqrt{RT_c}}{A\sqrt{P_c}\sqrt{P_c - P_e}} \quad ,$$

$$Re = \frac{\rho_c W D}{\mu_c} = \rho_c \left(\frac{P_c - P_e}{\rho_c}\right)^{\frac{1}{2}} \frac{D}{\mu} = \sqrt{\frac{P_c}{RT_c}}\sqrt{P_c - P_e}\,\frac{D}{\mu} \quad . \quad (33)$$

The quantity

$$W = \sqrt{\frac{P_c - P_e}{\rho_c}}$$

is defined by Liepmann as the characteristic velocity for orifice flow. The plot of Γ/Γ_K versus $1/Re$ is closely akin, but not identical, to

m/m_{fm} versus Knudsen number. Data from the present investigation are shown as the solidly-colored symbols for easy contrast with Liepmann's data, which are plotted for comparison. It is seen that the present data plot essentially in the middle of Liepmann's data down to a Knudsen number of about .01 . At this point, although there is a slight irregularity, perhaps corresponding to Liepmann's surmised "overshoot" due to boundary layer rounding off the orifice lip (p. 72 of ref. 8), the present data do not actually show a local maximum as do Liepmann's data. Instead, after a slight reflex in curvature, the present data extend smoothly toward an apparent continuum limit of Γ/Γ_K for lower Knudsen numbers.

If we divide equation (15), the continuum expression for mass flow through an orifice:

$$\dot{m} = \alpha\Gamma' \frac{P_c A}{\sqrt{RT_c}}$$

by equation (13), the free molecular expression:

$$\dot{m}_{fm} = \frac{\varkappa A(P_c - P_e)}{\sqrt{2\pi RT_c}} ,$$

we obtain:

$$\left.\frac{\dot{m}}{\dot{m}_{fm}}\right)_{Kn\to 0} = \frac{\alpha}{\varkappa}\Gamma'\sqrt{2\pi}\,\frac{P_c}{P_c - P_e} . \tag{34}$$

For both Liepmann's and the present investigation, the exhaust chamber pressure (P_e) is negligible compared with P_c , the stagnation chamber pressure. If we neglect P_e , equation (34) becomes:

$$\left.\frac{\dot{m}}{\dot{m}_{fm}}\right)_{Kn\to 0} = \frac{\alpha}{\varkappa}\Gamma'\sqrt{2\pi} .$$

This is the same as Liepmann's $\Gamma/\Gamma_{K\ 1/Re\to 0}$ since his $\Gamma_K = \varkappa/\ 2\pi$ and $\Gamma_{\ 1/Re\to 0} = \alpha\Gamma'$.

From Dushman[3] is obtained $\varkappa = .918$ for the orifice of $L/D = .0895$. For helium, $\gamma = 1.67$, and:

$$\Gamma' = \sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} = .725 \ .$$

Using Frankl's[9] $\alpha = .85$ for a two-dimensional slit, we finally obtain:

$$\left.\frac{\dot{m}}{\dot{m}_{fm}}\right)_{Kn\to 0} = \left.\frac{\Gamma}{\Gamma_K}\right)_{1/Re\to 0} = 1.682 \ ,$$

which is almost identical with the value for Γ/Γ_K at the lowest $1/Re$ ($1/Re = .000528$) shown in Figure 11. This is somewhat coincidental due to uncertainty concerning the value of α .

Figure 12 shows the same data from the present investigation plotted as m/m_{fm} versus Knudsen number and compared with Sreekanth's highest pressure ratio data (ref. 4), $P_c/P_e = 17$. Also shown is Sreekanth's semi-empirical formula for the transition regime, equation (23). It is seen that the present data agree quite well with Sreekanth's high pressure ratio data. Sreekanth's semi-empirical equation for m/m_{fm} as a function of Knudsen number appears to be a reasonable approximation to the data in the transition regime down to a Knudsen number of .2 , but begins to deviate from the data at lower Knudsen numbers.

V. CONCLUSIONS

The low-density gasdynamic facility described in this report is quite adequate for tests on small diameter nozzles, tubes, and orifices for the complete flow spectrum from continuum to free molecular. The pumping system is capable of maintaining at least a 100:1 pressure ratio across a 1-millimeter diameter test profile over the entire range of flow measurable by the installed flow measurement system. All flow rate and pressure measurements proved to be accurate within 1 per cent, as designed.

Tests conducted on an orifice, of length to diameter ratio .0895, show an asymptotic approach to a limiting value of the ratio of mass flow rate to theoretical free-molecule mass flow rate at both the lowest (Kn = .00068) and highest (Kn = 8.16) Knudsen number tests made, corresponding to a continuum limit and a free molecular limit, respectively. A smooth transition from one flow regime to the next is evident from the gradual change of the mass flow ratio with Knudsen number.

The current data and the data of Liepmann[8] and Sreekanth[4] all apparently show mass flow slightly greater than the theoretical free molecular rate at Knudsen number 10, previously considered well within the free molecule flow regime.

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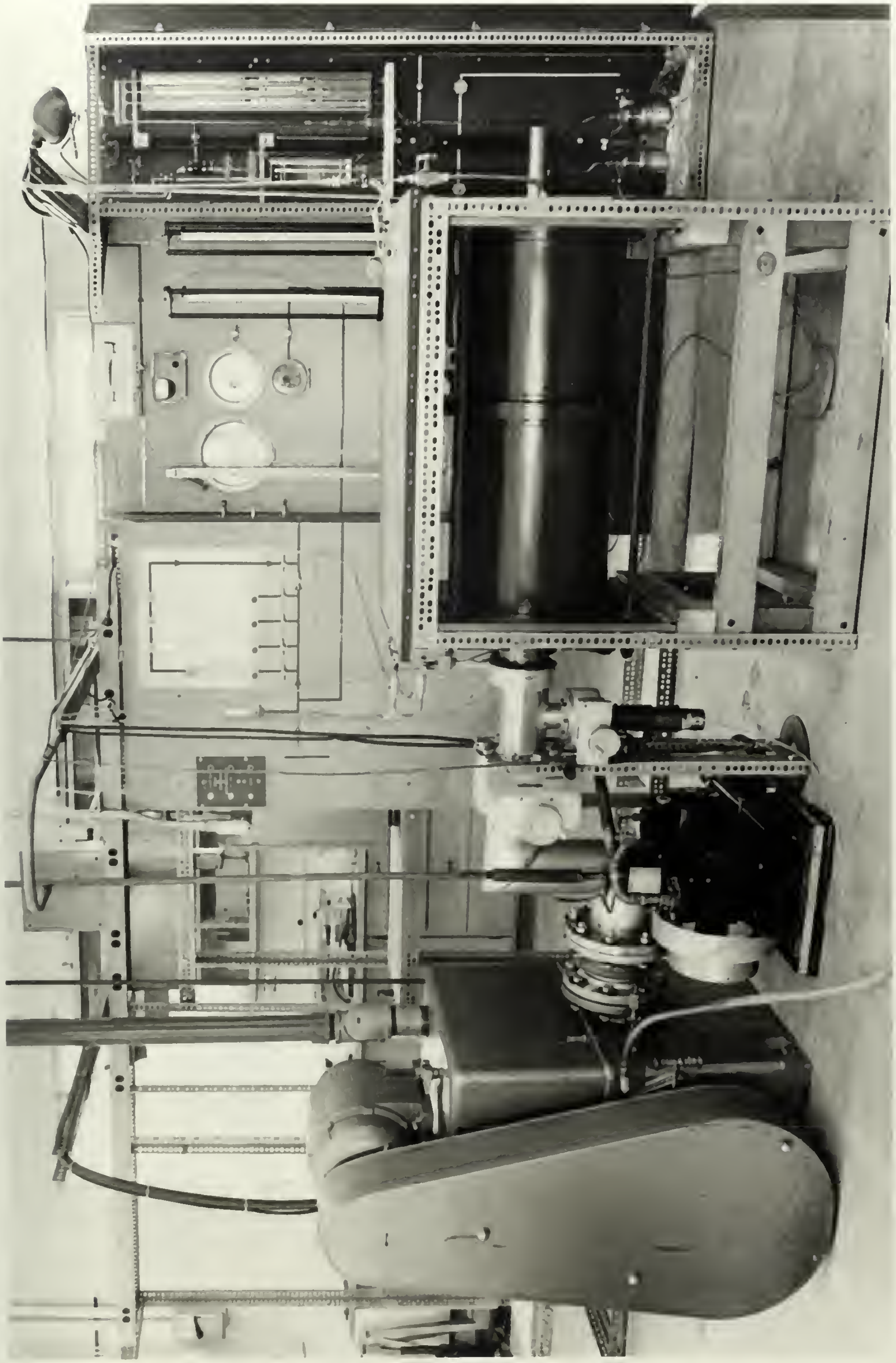
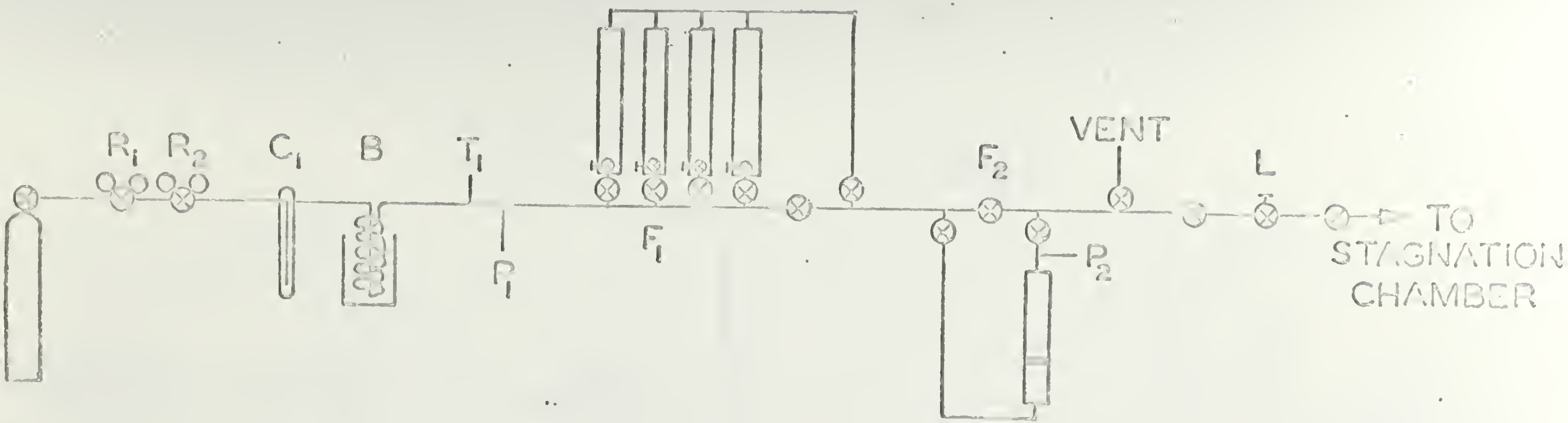
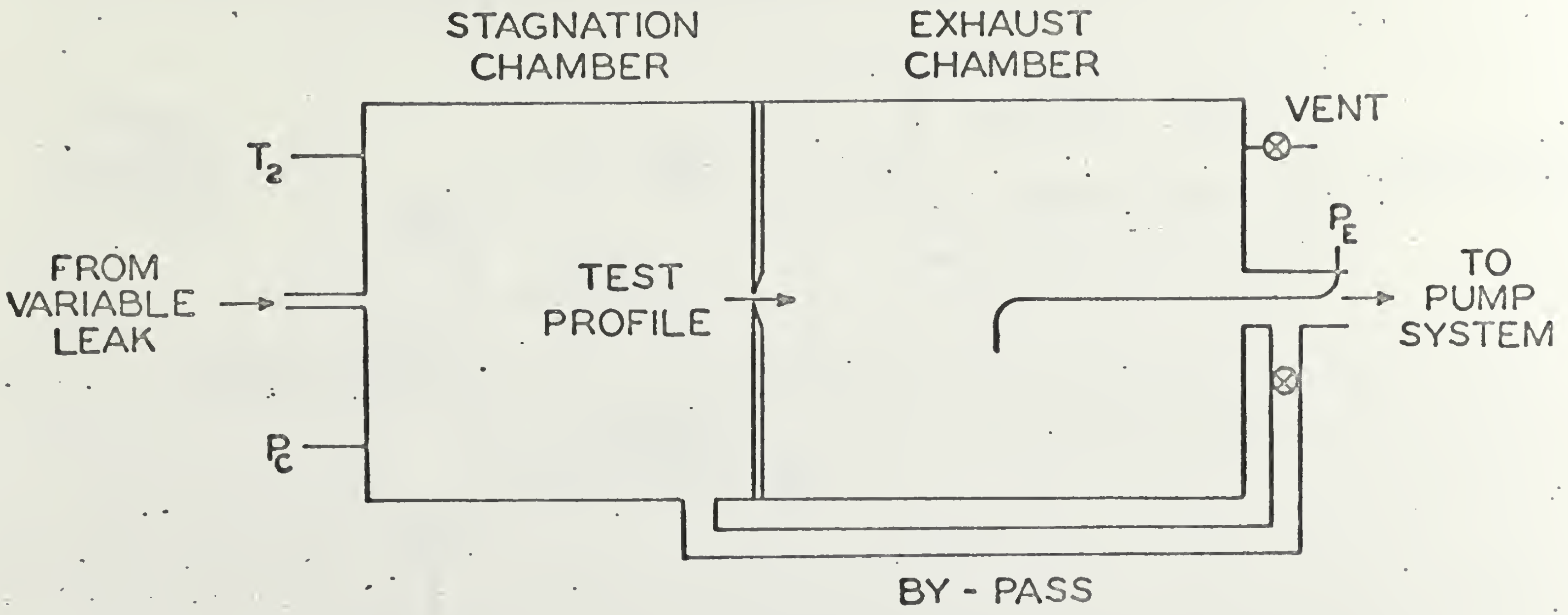


Figure 1. Low-Density Gasdynamic Facility.



R_1 - HIGH PRESSURE REGULATOR
R_2 - LOW PRESSURE REGULATOR
C_1 - LIQUID NITROGEN TRAP
B - CONSTANT TEMPERATURE BATH
T_1 - TEMPERATURE WELL
P_1 - PRESSURE TAP
F_1 - ROTAMETERS
F_2 - VOL-U-METER
P_2 - PRESSURE TAP
L - VARIABLE LEAK

FIG. 2 GAS FEED AND FLOW MEASUREMENT SYSTEM



T_2 - TEMPERATURE WELL

P_C - STAGNATION CHAMBER PRESSURE TAP

P_E - EXHAUST CHAMBER PRESSURE TAP

FIG. 3 VACUUM VESSEL SCHEMATIC

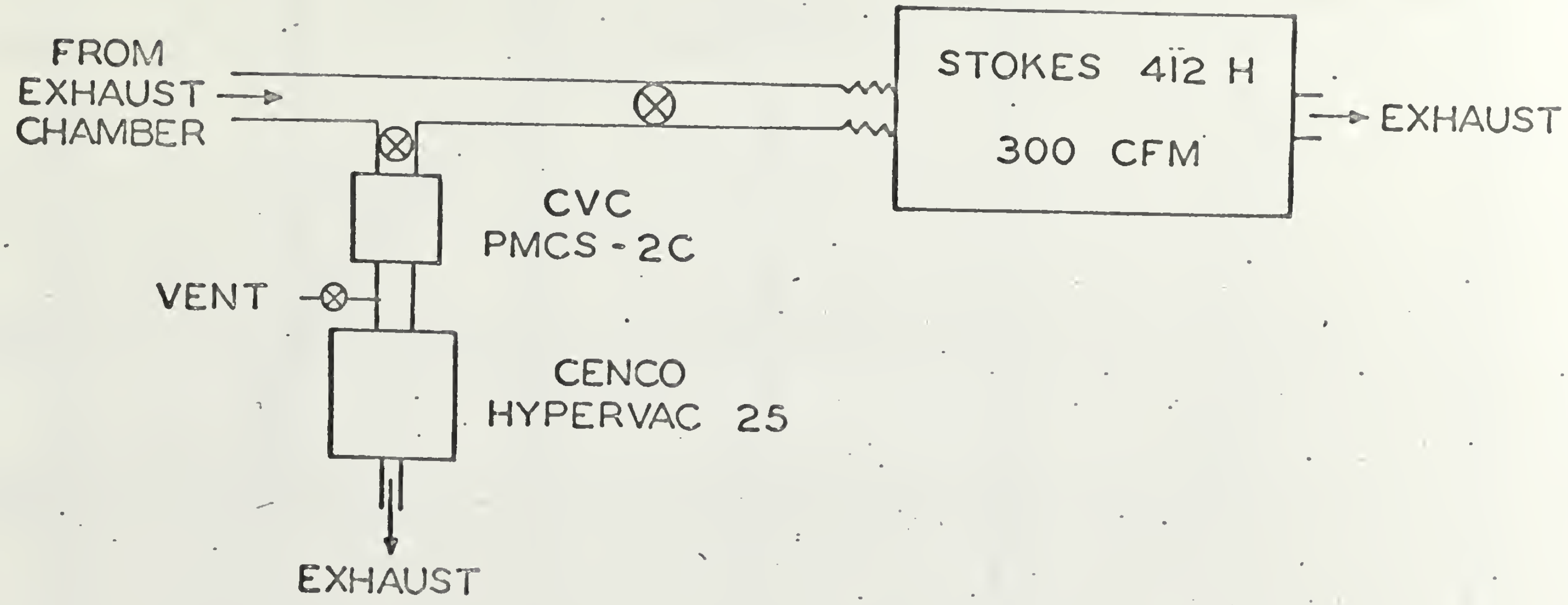


FIG. 4 PUMP SYSTEM SCHEMATIC

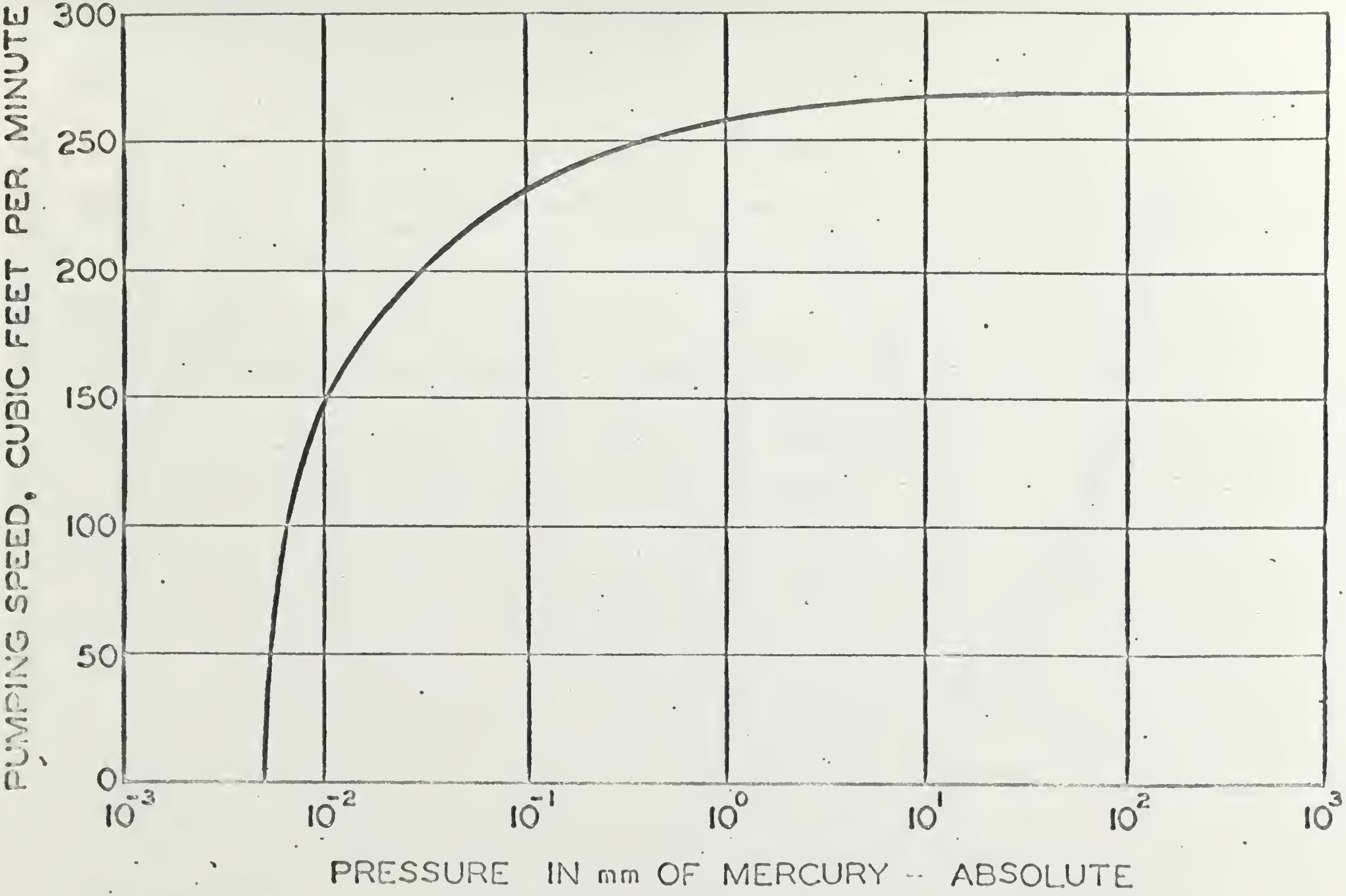


FIG. 5 STOKES 300 CFM DISPLACEMENT, 412-H PUMP

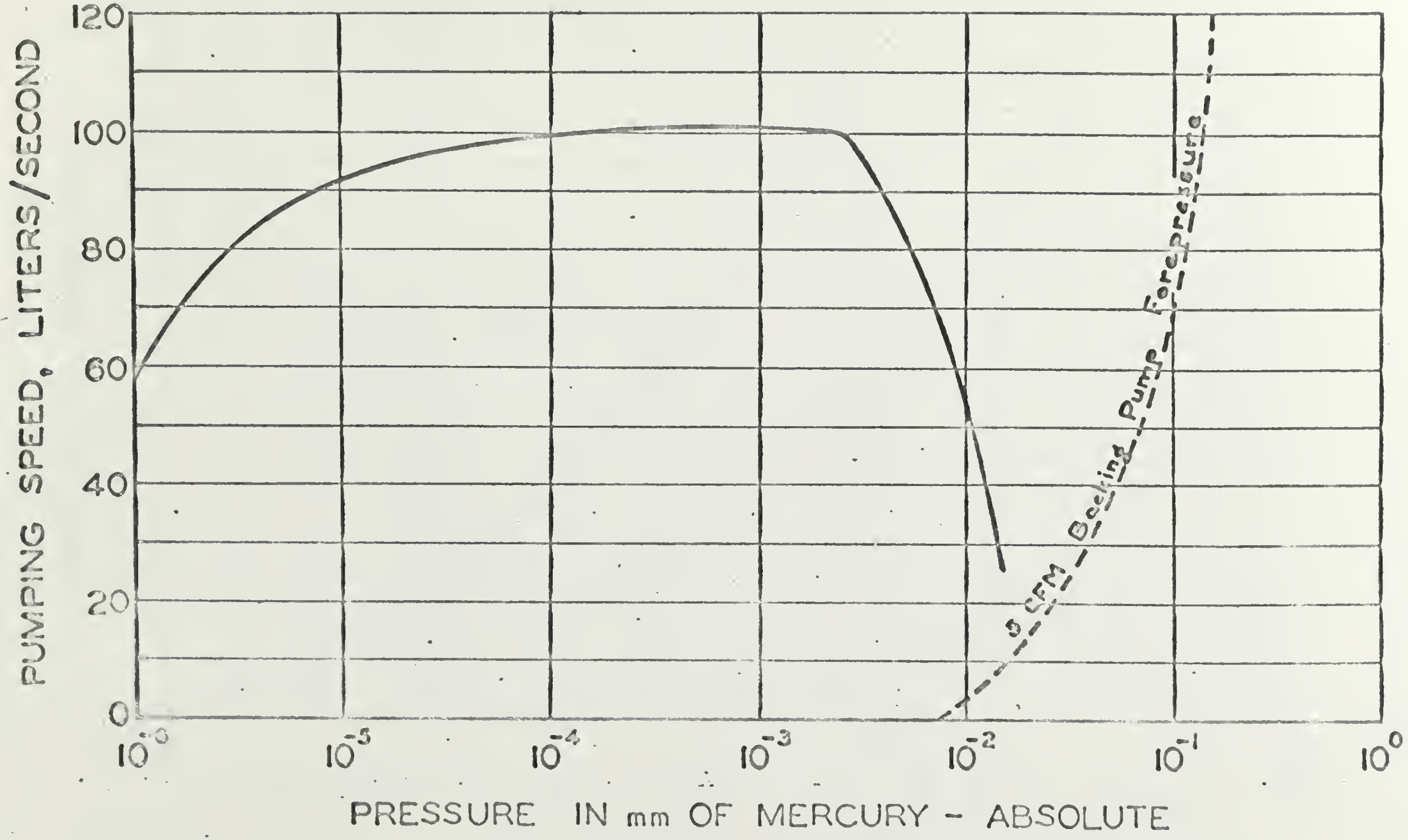


FIG. 6 CVC PMCS-2 FRACTIONATING DIFFUSION PUMP

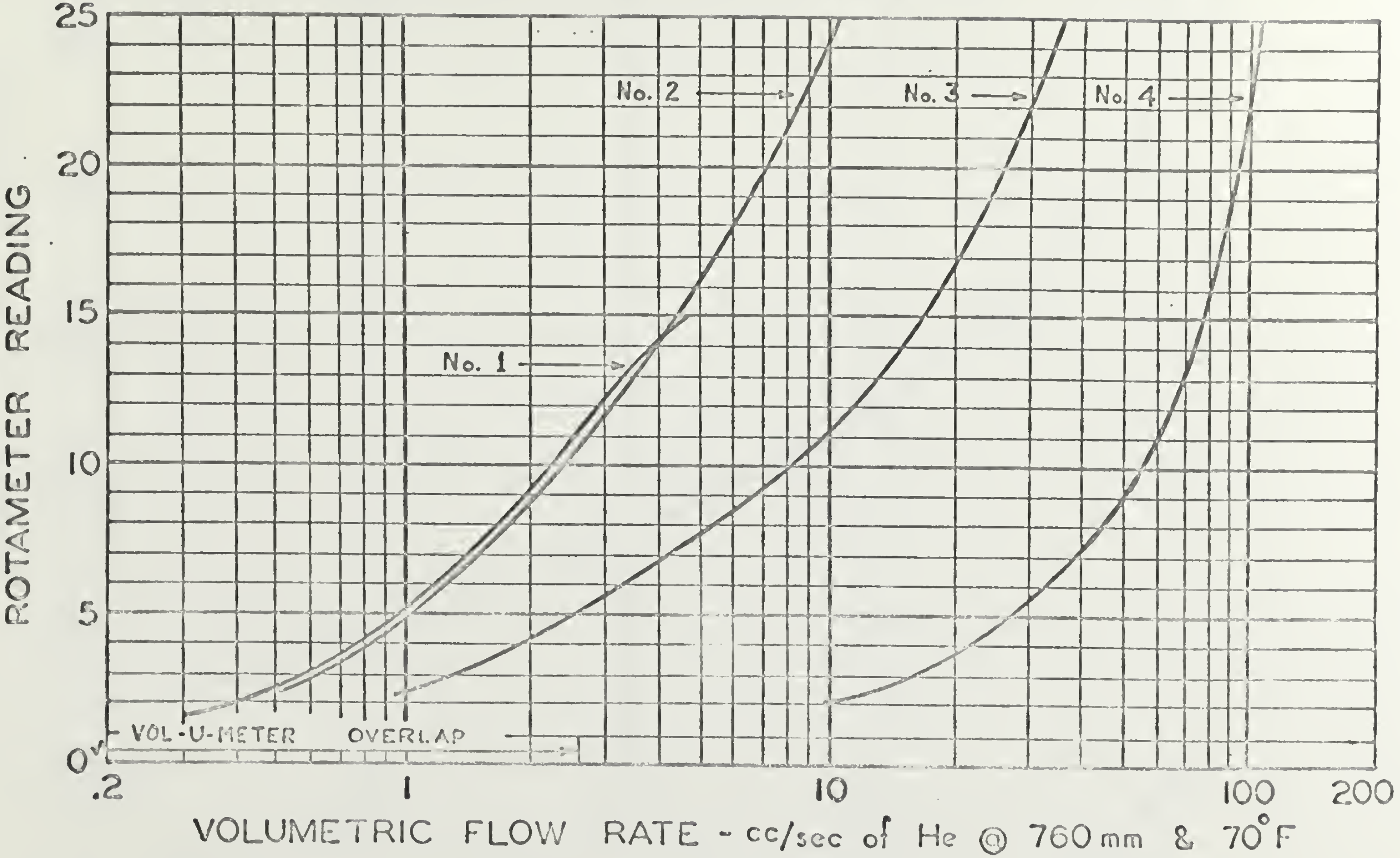


FIG. 7 ROTAMETER RANGES AND OVERLAP

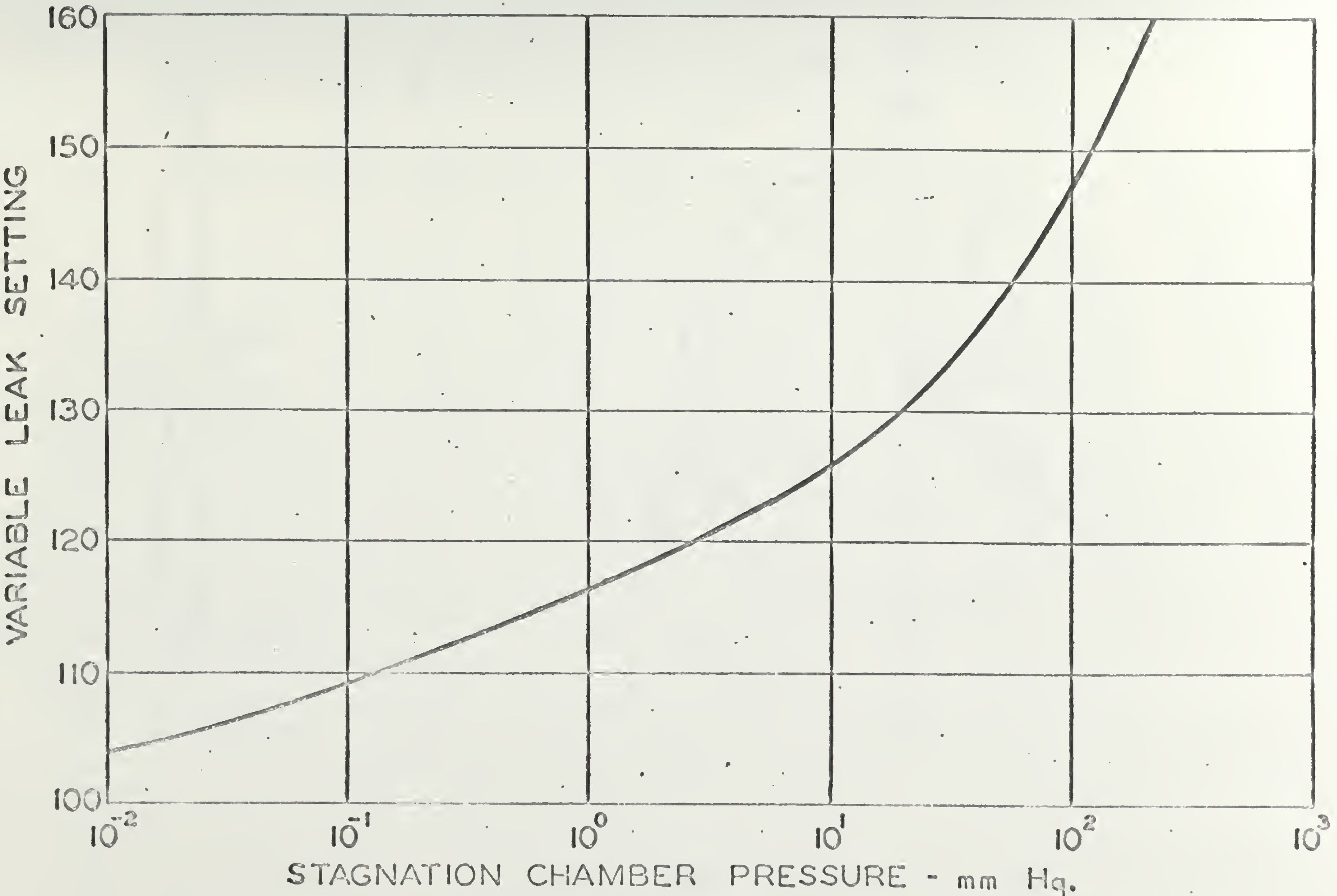


FIG. 8 VARIABLE LEAK SETTING VS. STAGNATION CHAMBER PRESSURE WITH ORIFICE TEST PROFILE

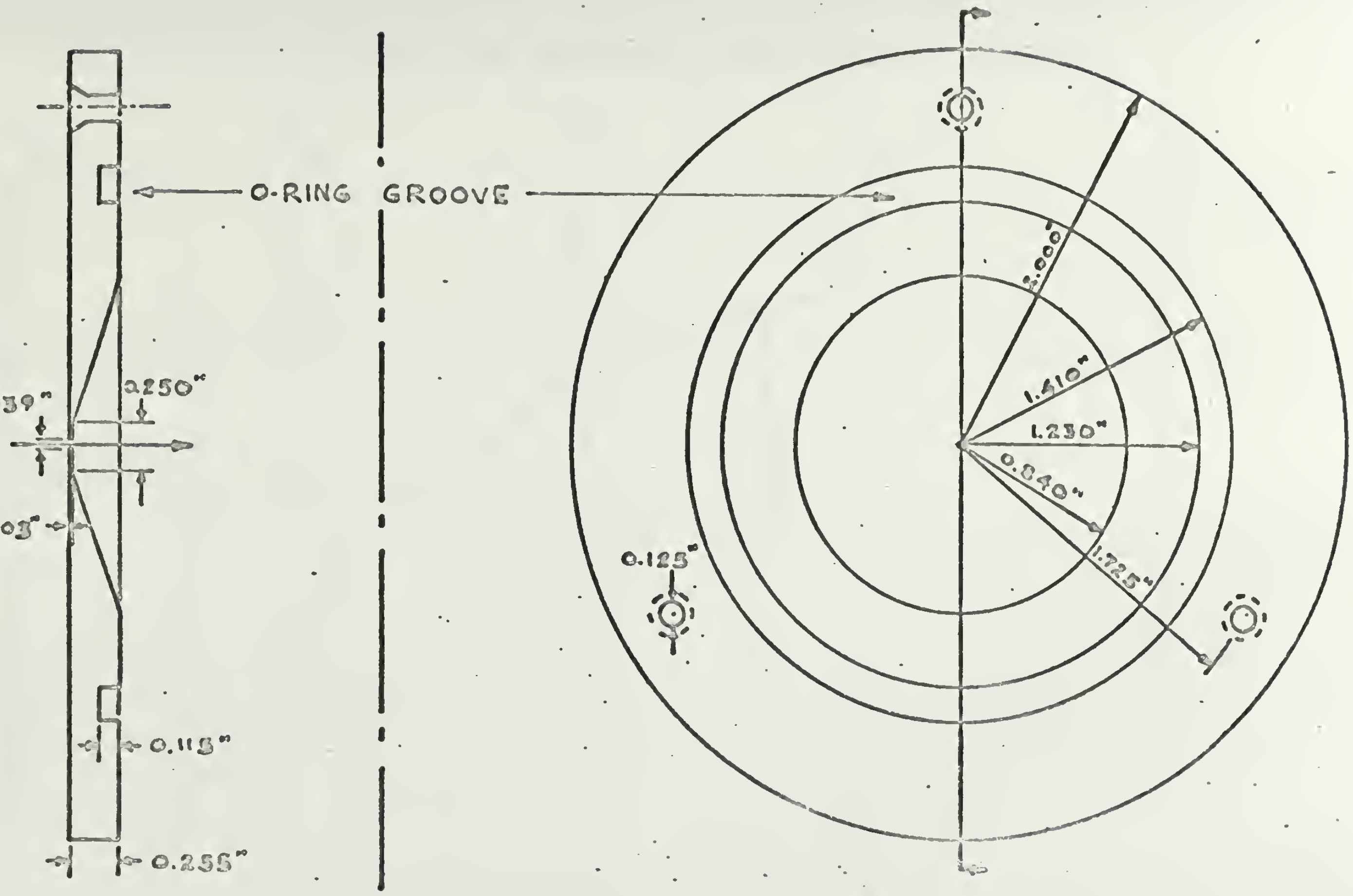


FIG. 9 TYPICAL TEST PROFILE PLATE (ORIFICE)

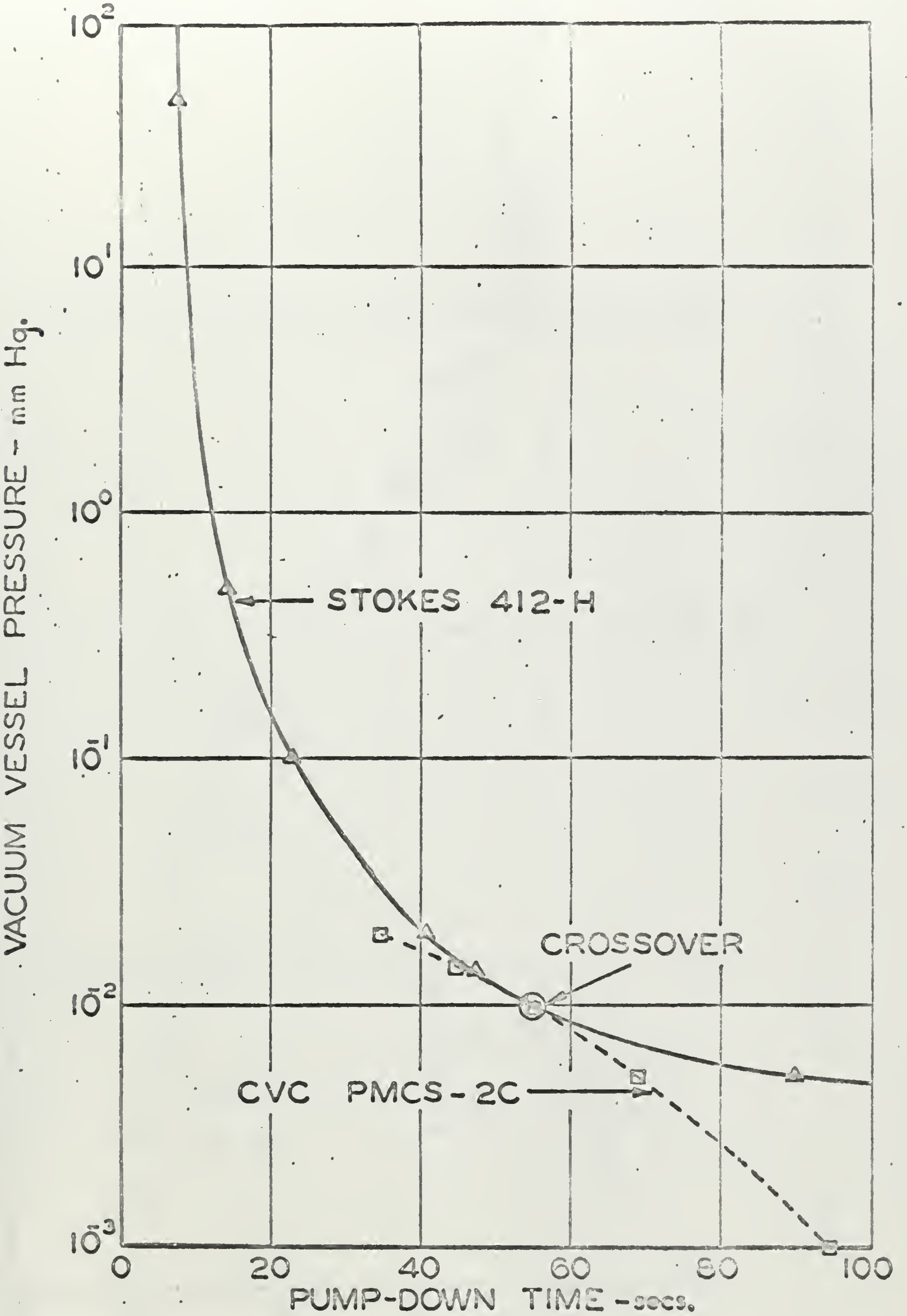


FIG. 10 SYSTEM PUMP-DOWN CHARACTERISTICS

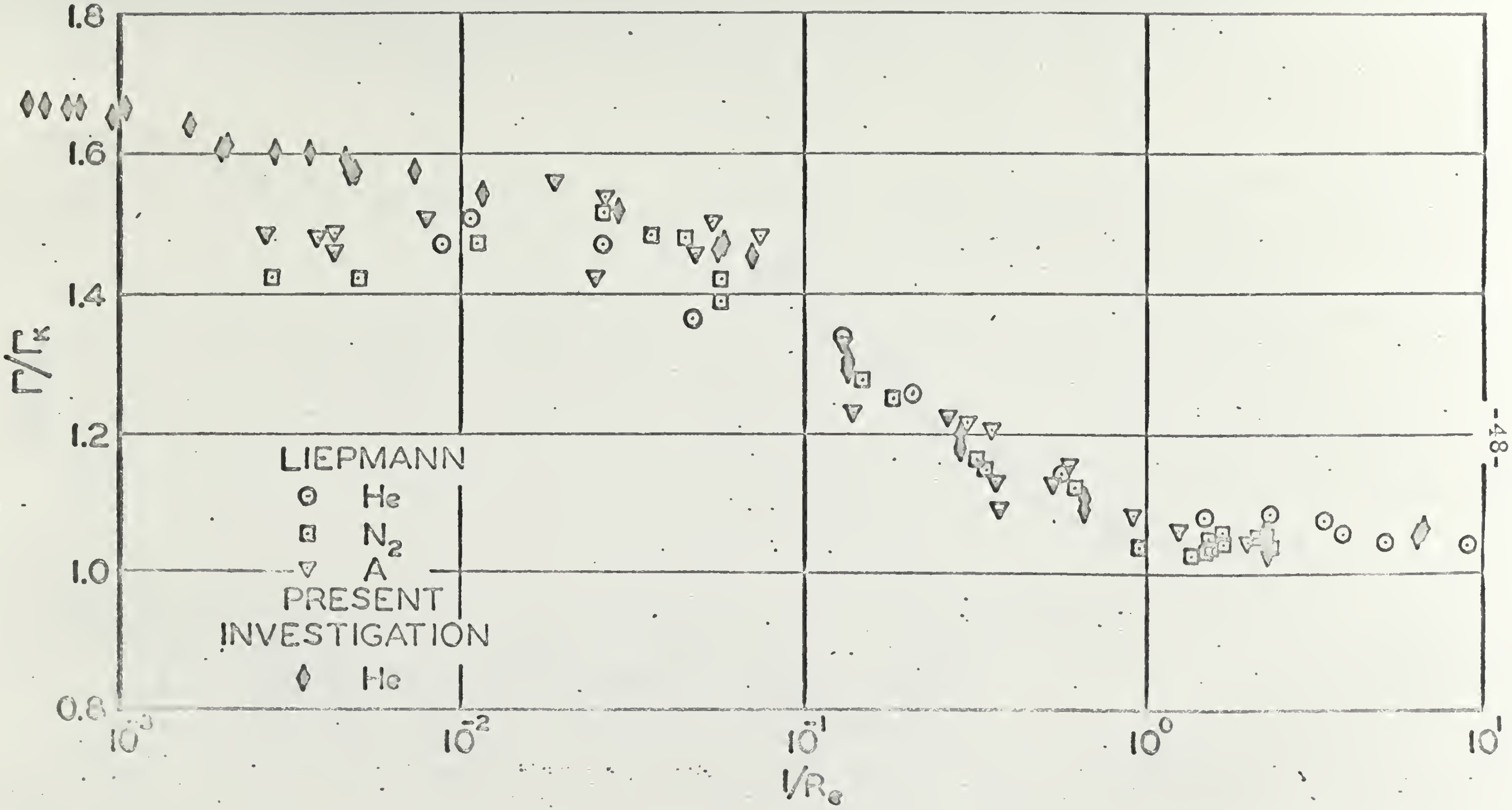


FIG. 11 Γ/Γ_K VS. $1/R_e$; Γ_K IS THE THEORETICAL ASYMTOTIC VALUE OF Γ FOR FREE MOLECULE FLOW

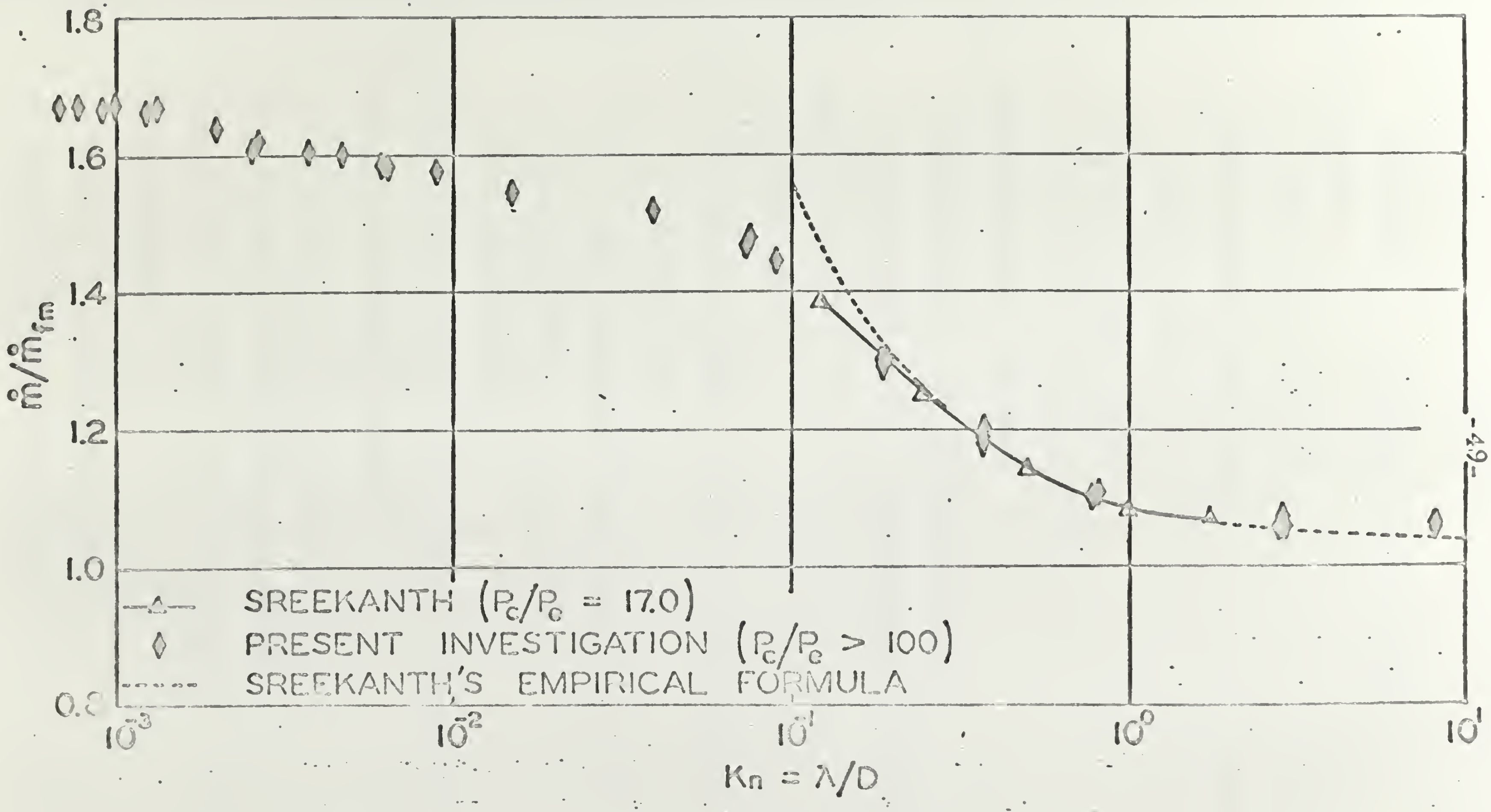


FIG. 12 NONDIMENSIONALIZED MASS FLOW VERSUS KNUDSEN NUMBER

APPENDIX I. Typical Test Procedure

I. Initial Pumpdown

A. Commencing with all valves closed:

1. Open shutoff valves to vacuum pressure measurement lines (3 valves)

2. Open tank bypass valve

3. Open 4" Temescal gate valve

B. Start cooling water flow to:

1. Stokes pump -- approximately 2 gal/min

2. CVC diffusion pump -- approximately 1/15 gal/min

C. Start all pumps:

1. Stokes pump

a. Check solenoid oil valve with ferromagnetic material

b. If not magnetized, shut down immediately

2. Cenco Hypervac 25 backing pump

3. CVC diffusion pump

D. Fill McLeod gauge cold-trap reservoir with liquid nitrogen

E. Open McLeod gauge stopcocks

F. Close variable leak to a counter setting of 010.

G. Open shutoff valve downstream of variable leak

H. When exhaust chamber pressure as read by thermistor no. 1 is 10 microns:

1. Close 4" Temescal gate valve

2. Open 2" Temescal gate valve

   3. Shut down Stokes pump

I. Pump down system to desired ultimate pressure. Note: if this is the first pumpdown after the system has been exposed to atmospheric conditions:

   1. Open Stokes pump gas ballast valves (2)
   2. Delete steps D and E
   3. Operate on gas ballast for 15 - 20 minutes
   4. Close gas ballast before going to Step H
   5. Pump down system for at least 24 hours before proceeding (it may be desirable to pump down for a few days to minimize outgassing)
   6. Execute steps D and E before proceeding

II. Test Gas Flow Setup

   A. Close tank bypass valve
   B. Close one of the shutoff valves to the McLeod gauge vacuum line (which one depends on whether exhaust or stagnation chamber pressure is to be measured)
   C. Fill feed line cold-trap reservoir with liquid nitrogen
   D. Start constant-temperature water bath
      1. Start motor and heater
      2. Set temperature regulator after flow rate is established
   E. Open all valves in the gas feed line except:
      1. Close individual rotameter shutoff valves
      2. Close rotameter outlet valve
      3. Close feed line shutoff valve immediately downstream of the feed line vent valve

F. Open helium (gas supply) bottle

   1. Set high-pressure regulator to about 100 psi (must be below 200 psi)

   2. Set low-pressure regulator to about 5" H_2O

G. After a few seconds of venting feed gas to the atmosphere:

   1. Close feed-line vent valve

   2. Open feed-line shutoff valve downstream of the vent

H. Open variable leak to the desired counter reading (graph of approximate counter reading versus stagnation pressure is helpful for this)

   1. Monitor exhaust chamber pressure on thermistor no. 1

   2. If exhaust chamber pressure exceeds 10 microns, switch pumps

      a. Close 2" Temescal gate valve

      b. Start Stokes pump and wait 30 seconds

      c. Open 4" Temescal gate valve

I. When satisfied with pump setup, monitor stagnation chamber pressure on thermistor no. 2, 0 - 50 mm dial gauge, or manometer. Note: Keep manometer shutoff valve closed below 50 mm Hg to minimize mercury vapor contamination of the variable leak.

III. Flow Rate Measurement

A. Low Flow Rate -- Vol-U-Meter

   1. Open input valve to rotameter no. 1

   2. Open rotameter output-line shutoff valve

3. Close rotameter bypass valve
4. Partially open rotameter no. 1 needle valve to give about .1 psi pressure drop between flow measurement system input pressure (0 - 15 psia dial gauge) and Vol-U-Meter output pressure (0 - 25, 25 - 50 psia dial gauge)
5. Stabilize system for at least 2 hours
6. Continually refill cold traps
7. Close Vol-U-Meter bypass valve and simultaneously open rotameter bypass valve. Note: this procedure minimizes changes in the gas input pressure to the variable leak when the Vol-U-Meter is thrown into the system.
8. Time the piston between any two volume marks (preferably 0 to 25 cc at the slower rates, and 0 and 22 cc at the faster rates)
9. Before the piston reaches the top of the tube:
    a. Stop the piston by cracking open the Vol-U-Meter bypass valve
    b. Lower the piston gently to the bottom of the tube by carefully opening the Vol-U-Meter bypass valve a bit more

B. Moderate to High Flow Rates -- Rotameters

1. Close input needle valve of desired rotameter
2. Open input valve to rotameter

3. Open rotameter output-line shutoff valve
4. Close rotameter bypass valve
5. Open rotameter input needle valve carefully to full open position. Note: this should be done carefully to prevent damage to the rotameter float.
6. The rotameter can be read continuously and will help indicate when the flow is stabilized (the primary means of assuring stabilization is stagnation chamber pressure measurements).

IV. Other Data Measurements

- A. Atmospheric conditions
    1. Pressure - barometer (apply temperature/gravity correction to indicated Hg column height)
    2. Temperature - thermometer on barometer case
- B. Gas inlet conditions
    1. Pressure - before flow measurement system
        - a. Low-pressure regulator dial gauge (0 - 30 in. H_2O) - this pressure plus atmospheric should equal flow measurement system input pressure
        - b. Flow measurement system input dial gauge (0 - 15 psia)
        - c. Flow measurement system input manometer (0 - 800 mm Hg gauge)
    3. Temperature - before flow measurement system

C. Stagnation Chamber Conditions
  1. Pressure
    a. Thermistor no. 2
    b. McLeod gauges
    c. Dial gauge (0 - 50 mm Hg)
    d. Manometer (0 - 800 mm Hg) - the scale is already corrected for temperature and gravity - subtract scale reading from corrected barometer reading
  2. Temperature - thermometer is in protective shroud protruding from upstream end of tank

D. Exhaust Chamber Conditions - Pressure
  1. Thermistor no. 1
  2. McLeod gauges

V. McLeod Gauge Operation

A. Activate pressurization system
  1. Close individual gauge shutoff valves
  2. Open vent line valve
  3. Open needle control valve
  4. Open nitrogen bottle valve
  5. Open regulator shutoff valve
  6. Set regulator pressure at approximately 25 psig
  7. Close needle control valve

B. Sample measurement
  1. Capture of gas sample (either gauge)
    a. Close vent line valve
    b. Open desired gauge shutoff valve 1 3/4 turns

    c. Open needle valve slowly to obtain rate of mercury rise such that sample capture takes 15 - 30 seconds

2. Non-linear gauge
    a. After sample capture, increase rate of mercury rise but slow it drastically near the top of the large volume to avoid shocking the closed capillary tube
    b. When mercury is in the closed capillary, increase the rate of rise again
    c. Slow the mercury rise as the level in the open capillary nears the top
    d. Stop the top of the mercury meniscus at the bottom of the black tape line by either closing the needle valve or closing the gauge shutoff valve
    e. Tap both capillaries a few times to overcome surface tension effects
    f. Read the difference in mercury column heights
3. Linear gauge
    a. After sample capture, increase rate of mercury rise but slow it near the top of each large volume to avoid overshooting the compression volume calibration mark
    b. Stop the mercury meniscus at the desired scribed volume mark by closing either the gauge shutoff

valve or the needle valve

(1) Use 1st scribe mark and left tube for pressures between 125 and 10 mm

(2) Use 2nd scribe mark and middle tube for pressures between 10 and 1 mm

(3) Use 3rd scribe mark and right tube for pressures between 1 and .1 mm (pressures below .15 mm can also be read on the non-linear McLeod)

c. Read the difference in mercury column heights

4. To lower the mercury level in either gauge:

a. Close needle valve

b. Open vent line valve

c. Open gauge shutoff valve

Note: if a slug of mercury hangs up in the closed capillary of the non-linear gauge, carefully heat the tube with a fuel-rich (yellow) natural gas/oxygen flame until the slug vaporizes.

5. Conversion from scale reading to pressure

a. Apply temperature and gravity correction to all mercury column heights before using scale factors

b. Scale factors

(1) 0 - 100 micron gauge: $P_{(\mu)} = .2375\times10^{-2}h_{mm}^{2}$

(2) 0-1 mm gauge: $P_{(mm)} = 1.86\times10^{-3}h_{mm}$

(3) 0 - 10 mm gauge: $P_{(mm)} = 1.894\times10^{-2}h_{mm}$

(4) 0 - 100 mm gauge: $P_{(mm)} = 2.494\times10^{-1} h_{mm}$

VI. System Shutdown

A. Test gas feed

1. Close valve downstream of variable leak
2. Close feed-line shutoff valve downstream of feed line vent
3. Check Vol-U-Meter bypass and rotameter bypass valves open
4. Turn off helium (test gas) bottle
5. Open feed-line vent valve

B. CVC Diffusion pump

1. Close 2" Temescal gate valve
2. Turn off diffusion-pump heater switch
3. Let water and Hypervac 25 run until heater feels cool to the touch
4. Turn off water
5. Turn off Hypervac 25

C. McLeod pressure system

1. Close both McLeod gauge shutoff valves
2. Open vent line valve
3. Turn off nitrogen bottle
4. Open needle control valve

VII. Periodic Checks

A. Check wooden shims under the test-tank support stand twice weekly and knock them back into positions marked on the

floor if vibration has moved them out.

B. Check the oil level in the Hypervac 25 backing pump at least every other day. Add Cenco HyVac oil while pump is running.

C. Oil level in the Stokes 412H pump should be halfway up the sight glass while the pump is running.

D. Temperature of diffusion-pump cooling water taken at the outlet should be 110 - 120 degrees Fahrenheit for maximum pumping rate. Check daily.

E. Stokes pump cooling water should be 70 - 100 degrees Fahrenheit at the outlet. Check during each period of extended operation.

F. Drain the water out of the Stokes pump and refill with clean water twice weekly.

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